**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Ans:** Find the probability that the service manager cannot meet the commitment means transmission service will take longer than 60 minutes since work start 10 min after car drop is X = 50.

µ = 45

σ = 8

Random variable X as the time required for servicing transmissions.

X > 50 minutes

Z =( X - µ)/σ

Calculate the Z-score for 50 minutes:

Z = (50 – 45)/8 = 5/8 = 0.625

For P(Z> 0.625), you subtract this probability from 1:

P(Z>0.625) = 1−P(Z<=0.625)

So answer is B.

**Refer to Set\_2\_Q1.ipynb**

1. 0.3875
2. **0.2676**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and **44.**

**Ans:** It is normal distribution. Majority of the employees are clustered around the mean.

µ = 38 σ = 6

Probability of employee > 44 = P(X > 44) = P(X<=44)

Z = ( X -µ)/σ =(44-38)/6

= P(Z<=1) in table 0.84134 = 84.134%

Probability employees will greater than 44 = 100 -84.134 =15.866

Probability of employees between 38 & 44:

= P(X<=44) – P (X>=38)

= P(Z >=(38 -38)/6) = P(Z>=0) = 0.5 table value

= 0.84134 -0.5 = 0.34134 = 34.134 %

So More employees at the processing center are older than 44 than between 38 and 44 is **True.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:** Probability of employee less than 30: P(x<30)

Z = (x-µ)/σ

= (30 -38)/6

= P(x<30) =P(z<(30 -38)/6) = P(z < -1.333)

Z table =.09176

The no. of employees with probability 0.0917 being under 30:

= 400 \*0.0917=36.68 =36

So training program for employees under the age of 30 at the center would be expected to attract about 36 employees is **True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

X1 ∼ N(µ, σ2 ), and X2 ∼ N(µ, σ2 ) are identically distributed two independent random variables difference between 2 X1 and X1 +X2

**Distribution of 2X1​ :**

Mean(µ’): E(2X1 ) = 2 . E(X1) = 2µ

Variance ( σ’^2): Var(2 X1 ) = (2 \* σ)2  = 4σ2

2X1​ normal distribution with mean 2μ and variance 4σ  
**Distribution of X1​+ X2 :**

Mean (μ"): E(X1​+ X2 ) = E(X1​) +E( X2 ) = µ +µ = 2µ

Variance (σ''^2): Var(X1​+ X2 ) = Var(X1​) +Var( X2 ) = σ2 + σ2 = 2 σ2

**X1​+ X2** normal distribution with mean 2μ and variance 2 σ2

As per above result 2X1 has wider spread compared to **X1​+ X2**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

**Ans:**

Probability of getting value between a and b should be: 0.99.

Probability outside the a and b area is : = 1-0.99 =0.01

Probability towards left from a = 0.01/2 = - 0.005

The Probability towards right from b = 0.01/2 = +0.005

So we need to calculate X, the random variable at a and b which has got these probabilities.

Standard Normal Variable Z:

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

= -(-2.57)\*20+100 = 151.4

= (-2.57)\*20+100 = 48.6

Option D is correct.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:** For a normal distribution, a 95% probability interval is typically given by approximately two standard deviations from the mean.

Confidence Interval=Mean±(Z-score × Standard Deviation)

In normal distribution table 95% confidence interval, the Z-score is approximately 1.96.

**Profit1**:

Mean=5×45

Standard Deviation=3×45

Confidence Interval=Mean±(1.96×Standard Deviation) = 225±(1.96 x 135)

upper and lower bounds of the confidence interval:

Lower bound=225−(1.96×135) = -39.6

Upper bound=225+(1.96×135) = 489.6

Rupee range (centered on the mean) for Profit1 with a 95% probability is approximately −39.6 to 489.6

**Profit2:**

Mean=7×45

Standard Deviation=4×45

Confidence Interval=Mean±(1.96×Standard Deviation) = 315 ± (1.96 x 180)

upper and lower bounds of the confidence interval:

Lower bound=315−(1.96×180) = -37.8

Upper bound=315+(1.96×180) = 667.8

Rupee range (centered on the mean) for Profit2 with a 95% probability is approximately −37.8 to 667.8

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans:**

5th Percentile=Mean + (Z-score × Standard Deviation)

**Profit1:**

Mean=225

Standard Deviation= 135

Z-score=−1.645

5th Percentile=225+(−1.645×135) = 225−221.025

= 3.975

5th percentile for Profit1 is approximately 3.975 Rupees

**Profit2:**

Mean=315

Standard Deviation=180

Z-score=−1.645

5th Percentile=315+(−1.645×180) = 315−295.8

= 19.2

5th percentile for Profit2 is approximately 19.2 Rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:**

Probability of loss for Profit1 can be calculated using the standard normal distribution:

Probability of Loss = P(Z < −Mean/​Standard Deviation)

**Profit1:**

Probability of Loss = P(z<−225/135)

= 0.0477903522728147

**Profit2:**

Probability of Loss=P(z<−315/180​)

= 0.040059156863817086

As per above probability Profit1 has a larger probability of making a loss in a given year compared to Profit2.